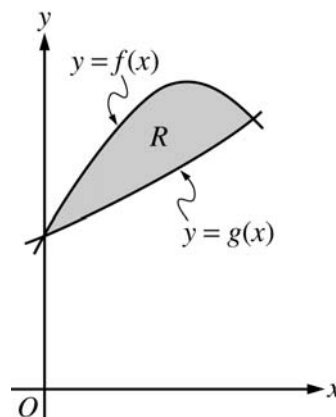


**AP<sup>®</sup> CALCULUS AB**  
**2005 SCORING GUIDELINES (Form B)**

**Question 1**

Let  $f$  and  $g$  be the functions given by  $f(x) = 1 + \sin(2x)$  and  $g(x) = e^{x/2}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$  as shown in the figure above.



- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
- (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles with diameters extending from  $y = f(x)$  to  $y = g(x)$ . Find the volume of this solid.

The graphs of  $f$  and  $g$  intersect in the first quadrant at  $(S, T) = (1.13569, 1.76446)$ .

$$\begin{aligned} \text{(a) Area} &= \int_0^S (f(x) - g(x)) \, dx \\ &= \int_0^S (1 + \sin(2x) - e^{x/2}) \, dx \\ &= 0.429 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^S ((f(x))^2 - (g(x))^2) \, dx \\ &= \pi \int_0^S ((1 + \sin(2x))^2 - (e^{x/2})^2) \, dx \\ &= 4.266 \text{ or } 4.267 \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= \int_0^S \frac{\pi}{2} \left( \frac{f(x) - g(x)}{2} \right)^2 \, dx \\ &= \int_0^S \frac{\pi}{2} \left( \frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 \, dx \\ &= 0.077 \text{ or } 0.078 \end{aligned}$$

1 : correct limits in an integral in (a), (b), or (c)

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ \langle -1 \rangle \text{ each error} \\ \text{Note: } 0/2 \text{ if integral not of form} \\ c \int_a^b (R^2(x) - r^2(x)) \, dx \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$