

AP[®] CALCULUS AB 2002 SCORING GUIDELINES

Question 3

An object moves along the x -axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

(a) $a(4) = v'(4) = \frac{\pi}{3} \cos\left(\frac{4\pi}{3}\right)$
 $= -\frac{\pi}{6}$ or -0.523 or -0.524

(b) On $3 < t < 4.5$:
 $a(t) = v'(t) = \frac{\pi}{3} \cos\left(\frac{\pi}{3}t\right) < 0$
 Statement I is correct since $a(t) < 0$.
 Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

(c) Distance = $\int_0^4 |v(t)| dt = 2.387$
 OR
 $x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$
 $x(0) = 2$
 $x(4) = 2 + \frac{9}{2\pi} = 3.43239$
 $v(t) = 0$ when $t = 3$
 $x(3) = \frac{6}{\pi} + 2 = 3.90986$
 $|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$

(d) $x(4) = x(0) + \int_0^4 v(t) dt = 3.432$
 OR
 $x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$
 $x(4) = 2 + \frac{9}{2\pi} = 3.432$

1 : answer

3 $\left\{ \begin{array}{l} 1 : \text{I correct, with reason} \\ 1 : \text{II correct} \\ 1 : \text{reason for II} \end{array} \right.$

3 $\left\{ \begin{array}{l} 1 : \left\{ \begin{array}{l} \text{limits of 0 and 4 on an integral} \\ \text{of } v(t) \text{ or } |v(t)| \\ \text{or} \\ \text{uses } x(0) \text{ and } x(4) \text{ to compute} \\ \text{distance} \end{array} \right. \\ 1 : \text{handles change of direction at} \\ \text{student's turning point} \\ 1 : \text{answer} \\ 0/1 \text{ if incorrect turning point or} \\ \text{no turning point} \end{array} \right.$

2 $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

2 $\left\{ \begin{array}{l} \text{OR} \\ 1 : x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + C \\ 1 : \text{answer} \\ 0/1 \text{ if no constant of integration} \end{array} \right.$