AP® CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 4

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) $a(t) = v'(t) = -e^{1-t}$
$a(3) = -e^{-2}$

$2 : \begin{cases} 1 : v'(t) \\ 1 : a(3) \end{cases}$

(b) $a(3) < 0$
$v(3) = -1 + e^{-2} < 0$
Speed is increasing since $v(3) < 0$ and $a(3) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $1 = e^{1-t}$, so $t = 1$.

$v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.
Therefore, the particle changes direction at $t = 1$.

$2 : \begin{cases} 1 : \text{solves } v(t) = 0 \text{ to get } t = 1 \\ 1 : \text{justifies change in direction at } t = 1 \end{cases}$

(d) Distance $= \int_0^3 |v(t)|\,dt$
$= \int_0^1 \left(-1 + e^{1-t}\right) dt + \int_1^3 \left(1 - e^{1-t}\right) dt$
$= \left(-t - e^{1-t}\Big|_0^1\right) + \left(t + e^{1-t}\Big|_1^3\right)$
$= (-1 - 1 + e) + \left(3 + e^{-2} - 1 - 1\right)$
$= e + e^{-2} - 1$

$4 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{antidifferentiation} \\ 1 : \text{evaluation} \end{cases}$

OR

OR

$x(t) = -t - e^{1-t}$
$x(0) = -e$
$x(1) = -2$
$x(3) = -e^{-2} - 3$
Distance $= (x(1) - x(0)) + (x(1) - x(3))$
$= (-2 + e) + \left(1 + e^{-2}\right)$
$= e + e^{-2} - 1$

$4 : \begin{cases} 1 : \text{any antiderivative} \\ 1 : \text{evaluates } x(t) \text{ when } t = 0,\ 1,\ 3 \\ 1 : \text{evaluates distance between points} \\ 1 : \text{evaluates total distance} \end{cases}$