AP® CALCULUS AB
2003 SCORING GUIDELINES

Question 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

(a) $a(2) = v'(2) = 1.587$ or 1.588
$v(2) = -3\sin(2) < 0$
Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

$2 :$ { $1 :$ $a(2)$; $1 :$ speed decreasing with reason }

(b) $v(t) = 0$ when $\dfrac{t^2}{2} = \pi$
$t = \sqrt{2\pi}$ or 2.506 or 2.507
Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

$2 :$ { $1 :$ $t = \sqrt{2\pi}$ only; $1 :$ justification }

(c) Distance $= \displaystyle\int_0^3 |v(t)|\,dt = 4.333$ or 4.334

$3 :$ { $1 :$ limits; $1 :$ integrand; $1 :$ answer }

(d) $\displaystyle\int_0^{\sqrt{2\pi}} v(t)\,dt = -3.265$
$x\left(\sqrt{2\pi}\right) = x(0) + \displaystyle\int_0^{\sqrt{2\pi}} v(t)\,dt = -2.265$
Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.

$2 :$ { $1 :$ $\pm$ (distance particle travels while velocity is negative); $1 :$ answer }