

AP[®] CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 3

A particle moves along the x -axis so that its velocity v at time t , for $0 \leq t \leq 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

- (a) Find the acceleration of the particle at time $t = 4$.
 (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left?
 (c) Find the position of the particle at time $t = 2$.
 (d) Find the average speed of the particle over the interval $0 \leq t \leq 2$.

(a) $a(4) = v'(4) = \frac{5}{7}$

1 : answer

(b) $v(t) = 0$
 $t^2 - 3t + 3 = 1$
 $t^2 - 3t + 2 = 0$
 $(t-2)(t-1) = 0$
 $t = 1, 2$

3 : $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = 0 \\ 1 : \text{direction change at } t = 1, 2 \\ 1 : \text{interval with reason} \end{array} \right.$

$v(t) > 0$ for $0 < t < 1$
 $v(t) < 0$ for $1 < t < 2$
 $v(t) > 0$ for $2 < t < 5$

The particle changes direction when $t = 1$ and $t = 2$.
 The particle travels to the left when $1 < t < 2$.

(c) $s(t) = s(0) + \int_0^t \ln(u^2 - 3u + 3) du$
 $s(2) = 8 + \int_0^2 \ln(u^2 - 3u + 3) du$
 $= 8.368$ or 8.369

3 : $\left\{ \begin{array}{l} 1 : \int_0^2 \ln(u^2 - 3u + 3) du \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{array} \right.$

(d) $\frac{1}{2} \int_0^2 |v(t)| dt = 0.370$ or 0.371

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$