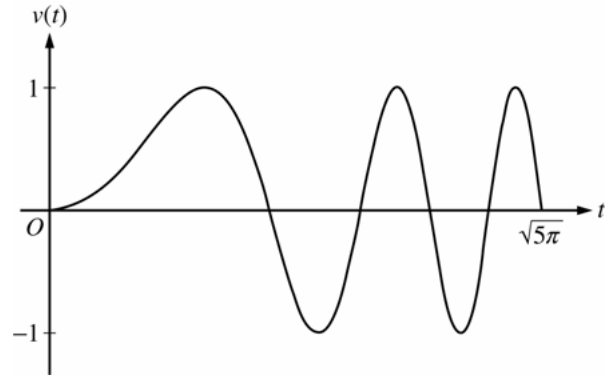


AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 2

A particle moves along the x -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



- (a) Find the acceleration of the particle at time $t = 3$.
 (b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.
 (c) Find the position of the particle at time $t = 3$.
 (d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

1 : $a(3)$

(b) Distance = $\int_0^3 |v(t)| dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and

$t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t) dt = 5.89483$

$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t) dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t) dt = 5.77356$

$|x(\sqrt{\pi}) - x(0)| + |x(\sqrt{2\pi}) - x(\sqrt{\pi})| + |x(3) - x(\sqrt{2\pi})| = 1.702$

2 : $\begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(c) $x(3) = 5 + \int_0^3 v(t) dt = 5.773$ or 5.774

3 : $\begin{cases} 2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ ($t = 1.772, 3.070, 3.963$).

Using $x(T) = 5 + \int_0^T v(t) dt$, the particle's positions at the times it changes from rightward to leftward movement are:

$T:$ 0 $\sqrt{\pi}$ $\sqrt{3\pi}$ $\sqrt{5\pi}$

$x(T):$ 5 5.895 5.788 5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

3 : $\begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$