

AP[®] CALCULUS AB
2007 SCORING GUIDELINES

Question 4

A particle moves along the x -axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.

- (a) Find the time t at which the particle is farthest to the left. Justify your answer.
 (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

- (a) $x'(t) = -e^{-t} \sin t + e^{-t} \cos t = e^{-t} (\cos t - \sin t)$
 $x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on
 $0 \leq t \leq 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.
 The candidates for the absolute minimum are at
 $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$, and 2π .

t	$x(t)$
0	$e^0 \sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}} \sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}} \sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi} \sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

- (b) $x''(t) = -e^{-t} (\cos t - \sin t) + e^{-t} (-\sin t - \cos t)$
 $= -2e^{-t} \cos t$
 $Ax''(t) + x'(t) + x(t)$
 $= A(-2e^{-t} \cos t) + e^{-t} (\cos t - \sin t) + e^{-t} \sin t$
 $= (-2A + 1)e^{-t} \cos t$
 $= 0$
 Therefore, $A = \frac{1}{2}$.

5 : $\left\{ \begin{array}{l} 2 : x'(t) \\ 1 : \text{sets } x'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{array} \right.$

4 : $\left\{ \begin{array}{l} 2 : x''(t) \\ 1 : \text{substitutes } x''(t), x'(t), \text{ and } x(t) \\ \quad \text{into } Ax''(t) + x'(t) + x(t) \\ 1 : \text{answer} \end{array} \right.$