

1998 Calculus AB Scoring Guidelines

2. Let f be the function given by $f(x) = 2xe^{2x}$.
- (a) Find $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow \infty} f(x)$.
- (b) Find the absolute minimum value of f . Justify that your answer is an absolute minimum.
- (c) What is the range of f ?
- (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b .

(a) $\lim_{x \rightarrow -\infty} 2xe^{2x} = 0$

$\lim_{x \rightarrow \infty} 2xe^{2x} = \infty$ or DNE

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$
if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
 $f'(x) > 0$ for all $x > -1/2$

–or–

(ii) $f'(x) \begin{array}{c} - \qquad \qquad + \\ \hline \qquad \qquad | \qquad \qquad \\ \qquad \qquad -1/2 \end{array}$

and $x = -1/2$ is the only critical number

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$
if $x = -1/b$
At $x = -1/b$, $y = -1/e$
 y has an absolute minimum value of $-1/e$ for all nonzero b

2 $\left\{ \begin{array}{l} 1: 0 \text{ as } x \rightarrow -\infty \\ 1: \infty \text{ or DNE as } x \rightarrow \infty \end{array} \right.$

3 $\left\{ \begin{array}{l} 1: \text{ solves } f'(x) = 0 \\ 1: \text{ evaluates } f \text{ at student's critical point} \\ \quad 0/1 \text{ if not local minimum from} \\ \quad \text{student's derivative} \\ 1: \text{ justifies absolute minimum value} \\ \quad 0/1 \text{ for a local argument} \\ \quad 0/1 \text{ without explicit symbolic} \\ \quad \text{derivative} \end{array} \right.$

Note: 0/3 if no absolute minimum based on student's derivative

1: answer

Note: must include the left-hand endpoint; exclude the right-hand "endpoint"

3 $\left\{ \begin{array}{l} 1: \text{ sets } y' = be^{bx}(1 + bx) = 0 \\ 1: \text{ solves student's } y' = 0 \\ 1: \text{ evaluates } y \text{ at a critical number} \\ \quad \text{and gets a value independent of } b \end{array} \right.$

Note: 0/3 if only considering specific values of b