

AP[®] CALCULUS AB
2001 SCORING GUIDELINES

Question 5

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a , b , and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b .

(b) If $\int_0^1 f(x) dx = 32$, what is the value of k ?

(a) $f'(x) = 12x^2 + 2ax + b$

$$f''(x) = 24x + 2a$$

$$f'(-1) = 12 - 2a + b = 0$$

$$f''(-2) = -48 + 2a = 0$$

$$a = 24$$

$$b = -12 + 2a = 36$$

$$5 : \left\{ \begin{array}{l} 1 : f'(x) \\ 1 : f''(x) \\ 1 : f'(-1) = 0 \\ 1 : f''(-2) = 0 \\ 1 : a, b \end{array} \right.$$

(b) $\int_0^1 (4x^3 + 24x^2 + 36x + k) dx$

$$= x^4 + 8x^3 + 18x^2 + kx \Big|_{x=0}^{x=1} = 27 + k$$

$$27 + k = 32$$

$$k = 5$$

$$4 : \left\{ \begin{array}{l} 2 : \text{antidifferentiation} \\ \quad < -1 > \text{ each error} \\ 1 : \text{expression in } k \\ 1 : k \end{array} \right.$$