

AP[®] CALCULUS AB
2001 SCORING GUIDELINES

Question 4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of h concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of h at $x = 4$.
- (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$$h'(x) \quad \begin{array}{ccccccc} & - & 0 & + & \text{und} & - & 0 & + \\ & & | & & | & & | & \\ x & & -\sqrt{2} & & 0 & & \sqrt{2} & \end{array}$$

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

(b) $h''(x) = 1 + \frac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

(c) $h'(4) = \frac{16 - 2}{4} = \frac{7}{2}$

$$y + 3 = \frac{7}{2}(x - 4)$$

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

$$4 : \left\{ \begin{array}{l} 1 : x = \pm\sqrt{2} \\ 1 : \text{analysis} \\ 2 : \text{conclusions} \\ \quad < -1 > \text{not dealing with} \\ \quad \text{discontinuity at } 0 \end{array} \right.$$

$$3 : \left\{ \begin{array}{l} 1 : h''(x) \\ 1 : h''(x) > 0 \\ 1 : \text{answer} \end{array} \right.$$

1 : tangent line equation

1 : answer with reason