

**AP<sup>®</sup> CALCULUS AB**  
**2003 SCORING GUIDELINES**

**Question 6**

Let  $f$  be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is  $f$  continuous at  $x = 3$ ? Explain why or why not.  
 (b) Find the average value of  $f(x)$  on the closed interval  $0 \leq x \leq 5$ .  
 (c) Suppose the function  $g$  is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$$

where  $k$  and  $m$  are constants. If  $g$  is differentiable at  $x = 3$ , what are the values of  $k$  and  $m$ ?

- (a)  $f$  is continuous at  $x = 3$  because

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = 2.$$

Therefore,  $\lim_{x \rightarrow 3} f(x) = 2 = f(3)$ .

(b) 
$$\begin{aligned} \int_0^5 f(x) dx &= \int_0^3 f(x) dx + \int_3^5 f(x) dx \\ &= \frac{2}{3}(x+1)^{3/2} \Big|_0^3 + \left(5x - \frac{1}{2}x^2\right) \Big|_3^5 \\ &= \left(\frac{16}{3} - \frac{2}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right) = \frac{20}{3} \end{aligned}$$

Average value:  $\frac{1}{5} \int_0^5 f(x) dx = \frac{4}{3}$

- (c) Since  $g$  is continuous at  $x = 3$ ,  $2k = 3m + 2$ .

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$$\lim_{x \rightarrow 3^-} g'(x) = \frac{k}{4} \text{ and } \lim_{x \rightarrow 3^+} g'(x) = m$$

Since these two limits exist and  $g$  is differentiable at  $x = 3$ , the two limits are equal. Thus  $\frac{k}{4} = m$ .

$$8m = 3m + 2; m = \frac{2}{5} \text{ and } k = \frac{8}{5}$$

2 :  $\left\{ \begin{array}{l} 1 : \text{answers "yes" and equates the} \\ \text{values of the left- and right-hand} \\ \text{limits} \\ 1 : \text{explanation involving limits} \end{array} \right.$

4 :  $\left\{ \begin{array}{l} 1 : k \int_0^3 f(x) dx + k \int_3^5 f(x) dx \\ \text{(where } k \neq 0) \\ 1 : \text{antiderivative of } \sqrt{x+1} \\ 1 : \text{antiderivative of } 5-x \\ 1 : \text{evaluation and answer} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : 2k = 3m + 2 \\ 1 : \frac{k}{4} = m \\ 1 : \text{values for } k \text{ and } m \end{array} \right.$