AB–4 1999

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
   (a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.
   (b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.
   (c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.
   (d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

(a) Slope at $x = 0$ is $f'(0) = -3$

At $x = 0$, $y = 2$

$y - 2 = -3(x - 0)$

1: equation

(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.

2 { 1: answer; 1: explanation }

(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$g'(0) = e^0(3f(0) + 2f'(0)) = 3(2) + 2(-3) = 0$$

$y - 4 = 0(x - 0)$

$y = 4$

2 { 1: $g'(0)$; 1: equation }

(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$g''(x) = (-2e^{-2x})(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x)) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$$

$g''(0) = e^0[(-6)(2) - (-3) + 2(0)] = -9$

Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$.

4 {
2: verify derivative
0/2 product or chain rule error
<−1> algebra errors
1: $g'(0) = 0$ and $g''(0)$
1: answer and reasoning }