

AP[®] CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 6

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

- (a) Find $f''(3)$.
- (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

(a) $f''(x) = \sqrt{f(x)} + x \cdot \frac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \frac{x^2}{2}$

$$f''(3) = \sqrt{25} + \frac{9}{2} = \frac{19}{2}$$

(b) $\frac{1}{\sqrt{y}} dy = x dx$

$$2\sqrt{y} = \frac{1}{2}x^2 + C$$

$$2\sqrt{25} = \frac{1}{2}(3)^2 + C; C = \frac{11}{2}$$

$$\sqrt{y} = \frac{1}{4}x^2 + \frac{11}{4}$$

$$y = \left(\frac{1}{4}x^2 + \frac{11}{4}\right)^2 = \frac{1}{16}(x^2 + 11)^2$$

3 : $\left\{ \begin{array}{l} 2 : f''(x) \\ < -2 > \text{ product or} \\ \text{chain rule error} \\ 1 : \text{ value at } x = 3 \end{array} \right.$

6 : $\left\{ \begin{array}{l} 1 : \text{ separates variables} \\ 1 : \text{ antiderivative of } dy \text{ term} \\ 1 : \text{ antiderivative of } dx \text{ term} \\ 1 : \text{ constant of integration} \\ 1 : \text{ uses initial condition } f(3) = 25 \\ 1 : \text{ solves for } y \end{array} \right.$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables