

AP[®] CALCULUS AB
2006 SCORING GUIDELINES

Question 6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.
- (b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(a) $g'(x) = ae^{ax} + f'(x)$
 $g'(0) = a - 4$

$g''(x) = a^2e^{ax} + f''(x)$
 $g''(0) = a^2 + 3$

$$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$$

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx)f(x)$
 $h'(0) = f'(0)\cos(0) - k\sin(0)f(0) = f'(0) = -4$
 $h(0) = \cos(0)f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$$5 : \begin{cases} 2 : h'(x) \\ 3 : \begin{cases} 1 : h'(0) \\ 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$