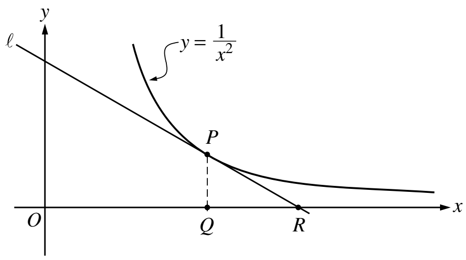
AB–6 1999



6. In the figure above, line ℓ is tangent to the graph of $y = \dfrac{1}{x^2}$ at point P, with coordinates $\left(w, \dfrac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x–axis at the point R, with coordinates $(k, 0)$.
   (a) Find the value of k when $w = 3$.
   (b) For all $w > 0$, find k in terms of w.
   (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?
   (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

(a) $\dfrac{dy}{dx} = -\dfrac{2}{x^3}$; $\left.\dfrac{dy}{dx}\right|_{x=3} = -\dfrac{2}{27}$

Line ℓ through $\left(3, \dfrac{1}{9}\right)$ and $(k, 0)$ has slope $-\dfrac{2}{27}$.

Therefore, $\dfrac{0 - \frac{1}{9}}{k - 3} = -\dfrac{2}{27}$ or $0 - \dfrac{1}{9} = -\dfrac{2}{27}(k - 3)$

$k = \dfrac{9}{2}$

$2\begin{cases} 1\text{: } \left.\dfrac{dy}{dx}\right|_{x=3} \\ 1\text{: answer} \end{cases}$

(b) Line ℓ through $\left(w, \dfrac{1}{w^2}\right)$ and $(k, 0)$ has slope $-\dfrac{2}{w^3}$.

Therefore, $\dfrac{0 - \frac{1}{w^2}}{k - w} = -\dfrac{2}{w^3}$ or $0 - \dfrac{1}{w^2} = -\dfrac{2}{w^3}(k - w)$

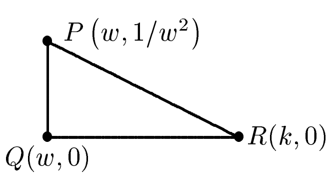
$k = \dfrac{3}{2}w$

$2\begin{cases} 1\text{: equation relating } w \text{ and } k\text{, using slopes} \\ 1\text{: answer} \end{cases}$

(c) $\dfrac{dk}{dt} = \dfrac{3}{2}\dfrac{dw}{dt} = \dfrac{3}{2} \cdot 7 = \dfrac{21}{2}$; $\left.\dfrac{dk}{dt}\right|_{w=5} = \dfrac{21}{2}$

1: answer using $\dfrac{dw}{dt} = 7$

(d)



$A = \dfrac{1}{2}(k - w)\dfrac{1}{w^2} = \dfrac{1}{2}\left(\dfrac{3}{2}w - w\right)\dfrac{1}{w^2} = \dfrac{1}{4w}$

$\dfrac{dA}{dt} = -\dfrac{1}{4w^2}\dfrac{dw}{dt}$

$\left.\dfrac{dA}{dt}\right|_{w=5} = -\dfrac{1}{100} \cdot 7 = -0.07$

Therefore, area is decreasing.

$4\begin{cases} 1\text{: area in terms of } w \text{ and/or } k \\ 1\text{: } \dfrac{dA}{dt} \text{ implicitly} \\ 1\text{: } \left.\dfrac{dA}{dt}\right|_{w=5} \text{ using } \dfrac{dw}{dt} = 7 \\ 1\text{: conclusion} \end{cases}$

Note: 0/4 if A constant