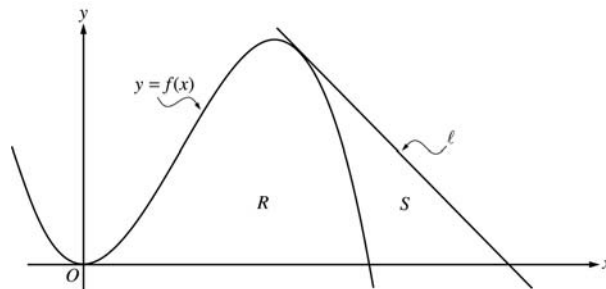


AP[®] CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 1

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f . Let R be the region bounded by the graph of f and the x -axis, and let S be the region bounded by the graph of f , the line ℓ , and the x -axis, as shown above.



- (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.
- (b) Find the area of S .
- (c) Find the volume of the solid generated when R is revolved about the x -axis.

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$
 $f(3) = 36 - 27 = 9$
 Tangent line at $x = 3$ is
 $y = -3(x - 3) + 9 = -3x + 18$,
 which is the equation of line ℓ .

(b) $f(x) = 0$ at $x = 4$
 The line intersects the x -axis at $x = 6$.
 Area = $\frac{1}{2}(3)(9) - \int_3^4 (4x^2 - x^3) dx$
 = 7.916 or 7.917
 OR

Area = $\int_3^4 ((18 - 3x) - (4x^2 - x^3)) dx$
 + $\frac{1}{2}(2)(18 - 12)$
 = 7.916 or 7.917

(c) Volume = $\pi \int_0^4 (4x^2 - x^3)^2 dx$
 = 156.038π or 490.208

1 : finds $f'(3)$ and $f(3)$
 2 : { finds equation of tangent line
 or
 1 : { shows $(3,9)$ is on both the
 graph of f and line ℓ

2 : integral for non-triangular region
 1 : limits
 4 : { 1 : integrand
 1 : area of triangular region
 1 : answer

3 : { 1 : limits and constant
 1 : integrand
 1 : answer