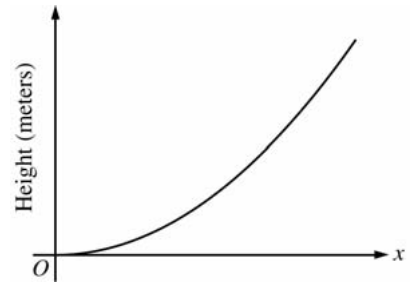


AP[®] CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 3

The figure above is the graph of a function of x , which models the height of a skateboard ramp. The function meets the following requirements.



- (i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.
 - (ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.
 - (iii) Between $x = 0$ and $x = 4$, the function is increasing.
- (a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.
- (b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.
- (c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.
- (d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.

(a) $f(4) = 1$ implies that $a = \frac{1}{16}$ and $f'(4) = 2a(4) = 1$
 implies that $a = \frac{1}{8}$. Thus, f cannot satisfy (ii).

2 : $\left\{ \begin{array}{l} 1 : a = \frac{1}{16} \text{ or } a = \frac{1}{8} \\ 1 : \text{shows } a \text{ does not work} \end{array} \right.$

(b) $g(4) = 64c - 1 = 1$ implies that $c = \frac{1}{32}$.
 When $c = \frac{1}{32}$, $g'(4) = 3c(4)^2 - \frac{2(4)}{16} = 3\left(\frac{1}{32}\right)(16) - \frac{1}{2} = 1$

1 : value of c

(c) $g'(x) = \frac{3}{32}x^2 - \frac{x}{8} = \frac{1}{32}x(3x - 4)$
 $g'(x) < 0$ for $0 < x < \frac{4}{3}$, so g does not satisfy (iii).

2 : $\left\{ \begin{array}{l} 1 : g'(x) \\ 1 : \text{explanation} \end{array} \right.$

(d) $h(4) = \frac{4^n}{k} = 1$ implies that $4^n = k$.
 $h'(4) = \frac{n4^{n-1}}{k} = \frac{n4^{n-1}}{4^n} = \frac{n}{4} = 1$ gives $n = 4$ and $k = 4^4 = 256$.

4 : $\left\{ \begin{array}{l} 1 : \frac{4^n}{k} = 1 \\ 1 : \frac{n4^{n-1}}{k} = 1 \\ 1 : \text{values for } k \text{ and } n \\ 1 : \text{verifications} \end{array} \right.$

$$h(x) = \frac{x^4}{256} \Rightarrow h(0) = 0.$$

$$h'(x) = \frac{4x^3}{256} \Rightarrow h'(0) = 0 \text{ and } h'(x) > 0 \text{ for } 0 < x < 4.$$