AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 4

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

(a) $f'(x) = 3\sqrt{4+(3x)^2}$

$g'(x) = f'(\sin x) \cdot \cos x$

$= 3\sqrt{4+(3\sin x)^2} \cdot \cos x$

$4 : \begin{cases} 2 : f'(x) \\ 2 : g'(x) \end{cases}$

(b) $g(\pi) = 0,\ g'(\pi) = -6$

Tangent line: $y = -6(x-\pi)$

$2 : \begin{cases} 1 : g(\pi) \text{ or } g'(\pi) \\ 1 : \text{tangent line equation} \end{cases}$

(c) For $0 < x < \pi,\ g'(x) = 0$ only at $x = \frac{\pi}{2}$.

$g(0) = g(\pi) = 0$

$g\left(\frac{\pi}{2}\right) = \int_0^3 \sqrt{4+t^2}\, dt > 0$

The maximum value of g on $[0, \pi]$ is $\int_0^3 \sqrt{4+t^2}\, dt$.

$3 : \begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{justifies maximum at } \frac{\pi}{2} \\ 1 : \text{integral expression for } g\left(\frac{\pi}{2}\right) \end{cases}$