

AP[®] CALCULUS AB
2007 SCORING GUIDELINES

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is rotated about the x -axis.
- (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

(a) $\text{Area} = \int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961 \text{ or } 37.962$

(b) $\text{Volume} = \pi \int_{-3}^3 \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$

(c) $\text{Volume} = \frac{\pi}{2} \int_{-3}^3 \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$
 $= \frac{\pi}{8} \int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in (a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$