1991 AB2
Solution

(a) $A = \int_0^1 1 + \sin(\pi x) - x^2 \, dx$

$$= \left(x - \frac{1}{\pi}\cos(\pi x) - \frac{1}{3}x^3 \right)\Big|_0^1$$

$$= \left(1 - \frac{1}{\pi}(-1) - \frac{1}{3} \right) - \left(0 - \frac{1}{\pi} - 0 \right)$$

$$= \frac{2}{3} + \frac{2}{\pi}$$

(b) $V = \pi \int_0^1 \left(1 + \sin(\pi x)\right)^2 - x^4 \, dx$

or

$$2\pi \int_0^1 y^{3/2} \, dy + 2\pi \int_1^2 y \left(1 - \frac{2}{\pi}\arcsin(y-1) \right) dy$$

(c) $V = 2\pi \int_0^1 x\left(1 + \sin(\pi x) - x^2\right) dx$

or

$$\pi \int_0^1 y \, dy + \pi \int_1^2 \left(1 - \frac{1}{\pi}\arcsin(y-1) \right)^2 - \left(\frac{1}{\pi}\arcsin(y-1) \right)^2 dy$$