1990 AB3
Solution

(a) $A = \int_0^1 e^x - (x-1)^2 \, dx$

$= \int_0^1 e^x - x^2 + 2x - 1 \, dx$

$= e^x \Big]_0^1 - \frac{1}{3}(x-1)^3 \Big]_0^1$

$= (e-1) - \frac{1}{3} = e - \frac{4}{3}$

(b) $V = \pi \int_0^1 e^{2x} - (x-1)^4 \, dx$

$= \pi \left[\frac{e^{2x}}{2} \right] \Bigg|_0^1 - \pi \left[\frac{1}{5}(x-1)^5 \right] \Bigg|_0^1$

$= \pi \left[\left(\frac{e^2}{2} - \frac{1}{2} \right) - \frac{1}{5} \right] = \pi \left(\frac{e^2}{2} - \frac{7}{10} \right)$

or

$V = 2\pi \int_0^1 y \left[1 - \left(1 - \sqrt{y}\right) \right] dy + 2\pi \int_1^e y(1 - \ln y) \, dy$

$= 2\pi \cdot \frac{2}{5} y^{5/2} \Big|_0^1 + 2\pi \left[\frac{1}{2} y^2 - \left(\frac{1}{2} y^2 \ln y - \frac{1}{4} y^2 \right) \right] \Bigg|_1^e$

$= \frac{4}{5}\pi + 2\pi \left[\frac{1}{4} e^2 - \frac{3}{4} \right] = \pi \left(\frac{e^2}{2} - \frac{7}{10} \right)$

(c) $V = 2\pi \int_0^1 x \left[e^x - (x-1)^2 \right] dx$

or

$V = \pi \int_0^1 1 - \left(1 - \sqrt{y}\right)^2 dy + \pi \int_1^e 1 - (\ln y)^2 \, dy$