

AP[®] CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- (a) How many gallons of heating oil are pumped into the tank during the time interval $0 \leq t \leq 12$ hours?
 (b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.
 (c) How many gallons of heating oil are in the tank at time $t = 12$ hours?
 (d) At what time t , for $0 \leq t \leq 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

(a) $\int_0^{12} H(t) dt = 70.570$ or 70.571

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(b) $H(6) - R(6) = -2.924$,
 so the level of heating oil is falling at $t = 6$.

1 : answer with reason

(c) $125 + \int_0^{12} (H(t) - R(t)) dt = 122.025$ or 122.026

3 : $\left\{ \begin{array}{l} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

(d) The absolute minimum occurs at a critical point or an endpoint.

$$H(t) - R(t) = 0 \text{ when } t = 4.790 \text{ and } t = 11.318.$$

3 : $\left\{ \begin{array}{l} 1 : \text{sets } H(t) - R(t) = 0 \\ 1 : \text{volume is least at} \\ \quad t = 11.318 \\ 1 : \text{analysis for absolute} \\ \quad \text{minimum} \end{array} \right.$

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$$125 + \int_0^{11.318} (H(t) - R(t)) dt = 120.738$$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.