1989 AB6
Solution

(a) $\dfrac{dy}{dt} = ky$ or $\begin{cases} \dfrac{dy}{y} = k\,dt \\ \ln|y| = kt + C_1 \\ y = e^{kt+C_1} \end{cases}$

$y = Ce^{kt}$

$t = 0 \Rightarrow C = 10^6,\ C_1 = \ln 10^6$

$\therefore y = 10^6 e^{kt}$

$t = 6 \Rightarrow \dfrac{1}{2} = e^{6k}$

$\therefore k = -\dfrac{\ln 2}{6}$

$y = 10^6 e^{\frac{-t}{6}\ln 2} = 10^6 \cdot 2^{\frac{-t}{6}}$

(b) $\dfrac{dy}{dt} = ky = -\dfrac{\ln 2}{6} \cdot 6 \cdot 10^5$

$= -10^5 \ln 2$

Decreasing at $10^5 \ln 2$ gal/year

(c) $5 \cdot 10^4 = 10^6 e^{kt}$

$\therefore kt = -\ln 20$

$\therefore t = \dfrac{-\ln 20}{\frac{-\ln 2}{6}}$

$= 6\dfrac{\ln 20}{\ln 2} = 6\log_2 20$

$6\dfrac{\ln 20}{\ln 2}$ years after starting