Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$\frac{dL}{dt} = \sqrt{t+1}$; $\quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$

$L(0) = 0$; $\quad C = -\frac{2}{3}$

$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}$; $\quad L(3) = \frac{14}{3}$

Scoring:

Method 1:

3 { 2 : definite integral (1 : limits; 1 : integrand); 1 : answer }

– or –

Method 2:

3 { 1 : antiderivative with C; 1 : solves for C using $L(0) = 0$; 1 : answer }

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx = 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

– or –

Method 2:

$\frac{dA}{dt} = 8 - \sqrt{t+1}$

$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$

$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C$; $\quad C = \frac{92}{3}$

$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$

Scoring:

Method 1:

2 { 1 : $30 + 8t$; 1 : $-\int_0^t \sqrt{x+1}\,dx$ }

– or –

Method 2:

2 { 1 : antiderivative with C; 1 : answer }

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

3 { 1 : sets $A'(t) = 0$; 1 : solves for t; 1 : justification }