

AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?
- (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
- (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$
 so the amount is not increasing at this time.

1 : answer with reason

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$
 $t = (5 \ln 3)^2 = 30.174$
 $P'(t)$ is negative for $0 < t < (5 \ln 3)^2$ and positive for $t > (5 \ln 3)^2$. Therefore there is a minimum at $t = (5 \ln 3)^2$.

3 { 1 : sets $P'(t) = 0$
 1 : solves for t
 1 : justification

(c) $P(30.174) = 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}}) dt$
 $= 35.104 < 40$, so the lake is safe.

3 { 1 : integrand
 1 : limits
 1 : conclusion with reason
 based on integral of $P'(t)$

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

2 { 1 : slope of tangent line
 1 : answer