

AP[®] CALCULUS AB
2001 SCORING GUIDELINES

Question 2

The temperature, in degrees Celsius ($^{\circ}\text{C}$), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ ($^{\circ}\text{C}$)
0	20
3	31
6	28
9	24
12	22
15	21

- (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
- (c) A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

- (a) Difference quotient; e.g.

$$W'(12) \approx \frac{W(15) - W(12)}{15 - 12} = -\frac{1}{3} \text{ }^{\circ}\text{C/day or}$$

$$W'(12) \approx \frac{W(12) - W(9)}{12 - 9} = -\frac{2}{3} \text{ }^{\circ}\text{C/day or}$$

$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = -\frac{1}{2} \text{ }^{\circ}\text{C/day}$$

- (b) $\frac{3}{2}(20 + 2(31) + 2(28) + 2(24) + 2(22) + 21) = 376.5$

$$\text{Average temperature} \approx \frac{1}{15}(376.5) = 25.1 \text{ }^{\circ}\text{C}$$

- (c) $P'(12) = 10e^{-t/3} - \frac{10}{3}te^{-t/3} \Big|_{t=12}$
 $= -30e^{-4} = -0.549 \text{ }^{\circ}\text{C/day}$

This means that the temperature is decreasing at the rate of $0.549 \text{ }^{\circ}\text{C/day}$ when $t = 12$ days.

- (d) $\frac{1}{15} \int_0^{15} (20 + 10te^{-t/3}) dt = 25.757 \text{ }^{\circ}\text{C}$

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer (with units)} \end{cases}$$

$$2 : \begin{cases} 1 : \text{trapezoidal method} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : P'(12) \text{ (with or without units)} \\ 1 : \text{interpretation} \end{cases}$$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and} \\ \quad \text{average value constant} \\ 1 : \text{answer} \end{cases}$$