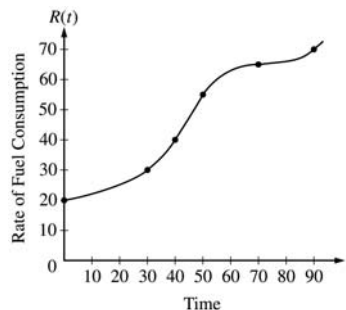


AP[®] CALCULUS AB
2003 SCORING GUIDELINES

Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- (c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- (d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a)
$$R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10}$$

$$= 1.5 \text{ gal/min}^2$$

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

(c)
$$\int_0^{90} R(t) dt \approx (30)(20) + (10)(30) + (10)(40)$$

$$+ (20)(55) + (20)(65) = 3700$$

Yes, this approximation is less because the graph of R is increasing on the interval.

- (d) $\int_0^b R(t) dt$ is the total amount of fuel in gallons consumed for the first b minutes.
 $\frac{1}{b} \int_0^b R(t) dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

2 : { 1 : a difference quotient using numbers from table and interval that contains 45
 1 : 1.5 gal/min²

2 : { 1 : $R''(45) = 0$
 1 : reason

2 : { 1 : value of left Riemann sum
 1 : "less" with reason

3 : { 2 : meanings
 1 : meaning of $\int_0^b R(t) dt$
 1 : meaning of $\frac{1}{b} \int_0^b R(t) dt$
 < - 1 > if no reference to time b
 1 : units in both answers