

AP[®] CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

(a) Midpoint Riemann sum is
 $10 \cdot [v(5) + v(15) + v(25) + v(35)]$
 $= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$
 The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 : $\begin{cases} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t .

2 : $\begin{cases} 1 : \text{two instances} \\ 1 : \text{justification} \end{cases}$

(c) $f'(23) = -0.407$ or -0.408 miles per minute²

1 : answer with units

(d) Average velocity $= \frac{1}{40} \int_0^{40} f(t) dt$
 $= 5.916$ miles per minute

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$