

**AP<sup>®</sup> CALCULUS AB  
2005 SCORING GUIDELINES**

**Question 3**

Distance $x$ (cm)	0	1	5	6	8
Temperature $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature  $T(x)$ , in degrees Celsius ( $^{\circ}\text{C}$ ), of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.

- (a) Estimate  $T'(7)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of  $T(x)$  for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find  $\int_0^8 T'(x) dx$ , and indicate units of measure. Explain the meaning of  $\int_0^8 T'(x) dx$  in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that  $T''(x) > 0$  for every  $x$  in the interval  $0 < x < 8$ ? Explain your answer.

(a) 
$$\frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{2} = -\frac{7}{2}^{\circ}\text{C/cm}$$

1 : answer

(b) 
$$\frac{1}{8} \int_0^8 T(x) dx$$

Trapezoidal approximation for  $\int_0^8 T(x) dx$ :

$$A = \frac{100 + 93}{2} \cdot 1 + \frac{93 + 70}{2} \cdot 4 + \frac{70 + 62}{2} \cdot 1 + \frac{62 + 55}{2} \cdot 2$$

Average temperature  $\approx \frac{1}{8} A = 75.6875^{\circ}\text{C}$

3 :  $\left\{ \begin{array}{l} 1 : \frac{1}{8} \int_0^8 T(x) dx \\ 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{array} \right.$

(c) 
$$\int_0^8 T'(x) dx = T(8) - T(0) = 55 - 100 = -45^{\circ}\text{C}$$

The temperature drops  $45^{\circ}\text{C}$  from the heated end of the wire to the other end of the wire.

2 :  $\left\{ \begin{array}{l} 1 : \text{value} \\ 1 : \text{meaning} \end{array} \right.$

(d) Average rate of change of temperature on  $[1, 5]$  is  $\frac{70 - 93}{5 - 1} = -5.75$ .

Average rate of change of temperature on  $[5, 6]$  is  $\frac{62 - 70}{6 - 5} = -8$ .

No. By the MVT,  $T'(c_1) = -5.75$  for some  $c_1$  in the interval  $(1, 5)$  and  $T'(c_2) = -8$  for some  $c_2$  in the interval  $(5, 6)$ . It follows that  $T'$  must decrease somewhere in the interval  $(c_1, c_2)$ . Therefore  $T''$  is not positive for every  $x$  in  $[0, 8]$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{two slopes of secant lines} \\ 1 : \text{answer with explanation} \end{array} \right.$

Units of  $^{\circ}\text{C/cm}$  in (a), and  $^{\circ}\text{C}$  in (b) and (c)

1 : units in (a), (b), and (c)