

AP[®] CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ (ft/sec ²)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v , measured in feet per second, and acceleration a , measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

- (a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)| dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)| dt$ using a trapezoidal approximation with the three subintervals determined by the table.
- (b) Using appropriate units, explain the meaning of $\int_0^{30} a(t) dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t) dt$.
- (c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.
- (d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

- (a) $\int_{30}^{60} |v(t)| dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)| dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

- (b) $\int_0^{30} a(t) dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\begin{aligned} \int_0^{30} a(t) dt &= \int_0^{30} v'(t) dt = v(30) - v(0) \\ &= -14 - (-20) = 6 \text{ ft/sec} \end{aligned}$$

- (c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

- (d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

Units of ft in (a) and ft/sec in (b)

2 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

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2 : $\begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$

2 : $\begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$

1 : units in (a) and (b)