

AP[®] CALCULUS AB
2007 SCORING GUIDELINES

Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

- (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$
 $\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2 2 = 7200\pi$ ft³/min

(c) $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
 $= 19.3$ ft
 $\int_0^{12} r'(t) dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t) dt$.

Units of ft³/min in part (b) and ft in part (c)

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

1 : conclusion with reason

1 : units in (b) and (c)