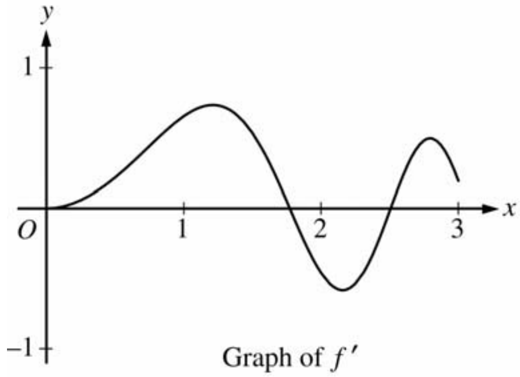
Question 2



Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)}\sin\left(x^2\right)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

$2: \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \ldots$

On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$f(0) = 5$

$f\left(\sqrt{\pi}\right) = f(0) + \int_0^{\sqrt{\pi}} f'(x)\,dx = 5.67911$

$f(3) = f(0) + \int_0^3 f'(x)\,dx = 5.57893$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

$3: \begin{cases} 1 : \text{identifies } \sqrt{\pi} \text{ and 3 as candidates} \\ \qquad \text{- or -} \\ \quad \text{indicates that the graph of } f \\ \quad \text{increases, decreases, then increases} \\ 1 : \text{justifies } f\left(\sqrt{\pi}\right) > f(3) \\ 1 : \text{answer} \end{cases}$

(c) $f(2) = f(0) + \int_0^2 f'(x)\,dx = 5.62342$

$f'(2) = e^{-0.5}\sin(4) = -0.45902$

$y - 5.623 = (-0.459)(x - 2)$

$4: \begin{cases} 2 : f(2) \text{ expression} \\ \qquad 1 : \text{integral} \\ \qquad 1 : \text{including } f(0) \text{ term} \\ 1 : f'(2) \\ 1 : \text{equation} \end{cases}$