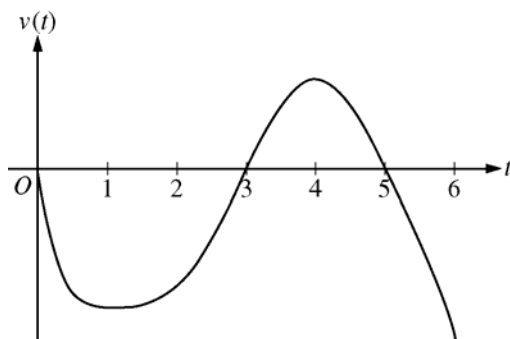


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**2008 SCORING GUIDELINES**

**Question 4**



Graph of  $v$

A particle moves along the  $x$ -axis so that its velocity at time  $t$ , for  $0 \leq t \leq 6$ , is given by a differentiable function  $v$  whose graph is shown above. The velocity is 0 at  $t = 0$ ,  $t = 3$ , and  $t = 5$ , and the graph has horizontal tangents at  $t = 1$  and  $t = 4$ . The areas of the regions bounded by the  $t$ -axis and the graph of  $v$  on the intervals  $[0, 3]$ ,  $[3, 5]$ , and  $[5, 6]$  are 8, 3, and 2, respectively. At time  $t = 0$ , the particle is at  $x = -2$ .

- (a) For  $0 \leq t \leq 6$ , find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of  $t$ , where  $0 \leq t \leq 6$ , is the particle at  $x = -8$ ? Explain your reasoning.
- (c) On the interval  $2 < t < 3$ , is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

- (a) Since  $v(t) < 0$  for  $0 < t < 3$  and  $5 < t < 6$ , and  $v(t) > 0$  for  $3 < t < 5$ , we consider  $t = 3$  and  $t = 6$ .

$$x(3) = -2 + \int_0^3 v(t) dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t) dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time  $t = 3$  when its position is  $x(3) = -10$ .

- (b) The particle moves continuously and monotonically from  $x(0) = -2$  to  $x(3) = -10$ . Similarly, the particle moves continuously and monotonically from  $x(3) = -10$  to  $x(5) = -7$  and also from  $x(5) = -7$  to  $x(6) = -9$ .

By the Intermediate Value Theorem, there are three values of  $t$  for which the particle is at  $x(t) = -8$ .

- (c) The speed is decreasing on the interval  $2 < t < 3$  since on this interval  $v < 0$  and  $v$  is increasing.
- (d) The acceleration is negative on the intervals  $0 < t < 1$  and  $4 < t < 6$  since velocity is decreasing on these intervals.

$$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t) dt \\ 1 : \text{conclusion} \end{cases}$$

$$3 : \begin{cases} 1 : \text{positions at } t = 3, t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$$

1 : answer with reason

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$