

AP[®] CALCULUS AB 2002 SCORING GUIDELINES

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

- (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.
- (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.
- (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$. Show the analysis that leads to your answers.

(a) Area = $\int_{\frac{1}{2}}^1 (e^x - \ln x) dx = 1.222$ or 1.223

2 { 1 : integral
1 : answer

(b) Volume = $\pi \int_{\frac{1}{2}}^1 ((4 - \ln x)^2 - (4 - e^x)^2) dx$
 = 7.515π or 23.609

4 { 1 : limits and constant
2 : integrand
< -1 > each error
Note: 0/2 if not of the form
 $k \int_a^b (R(x)^2 - r(x)^2) dx$
1 : answer

(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0$
 $x = 0.567143$

3 { 1 : considers $h'(x) = 0$
1 : identifies critical point
and endpoints as candidates
1 : answers

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$

$h(0.5) = 2.3418$

$h(1) = 2.718$

The absolute minimum is 2.330.

The absolute maximum is 2.718.

Note: Errors in computation come off the third point.