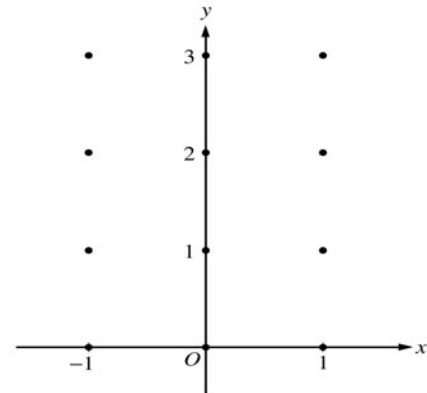


**AP<sup>®</sup> CALCULUS AB  
2004 SCORING GUIDELINES**

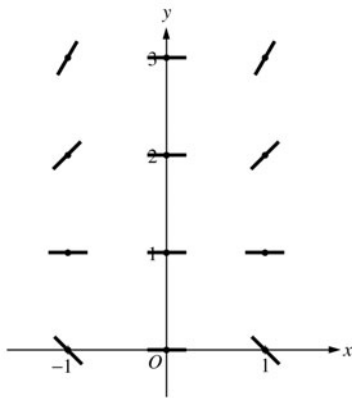
**Question 6**

Consider the differential equation  $\frac{dy}{dx} = x^2(y - 1)$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
(Note: Use the axes provided in the pink test booklet.)
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane. Describe all points in the  $xy$ -plane for which the slopes are positive.
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 3$ .



(a)



- (b) Slopes are positive at points  $(x, y)$  where  $x \neq 0$  and  $y > 1$ .

(c)  $\frac{1}{y-1} dy = x^2 dx$

$$\ln|y-1| = \frac{1}{3}x^3 + C$$

$$|y-1| = e^C e^{\frac{1}{3}x^3}$$

$$y-1 = Ke^{\frac{1}{3}x^3}, K = \pm e^C$$

$$2 = Ke^0 = K$$

$$y = 1 + 2e^{\frac{1}{3}x^3}$$

- 1 : zero slope at each point  $(x, y)$  where  $x = 0$  or  $y = 1$
- 2 : { positive slope at each point  $(x, y)$  where  $x \neq 0$  and  $y > 1$
- 1 : { negative slope at each point  $(x, y)$  where  $x \neq 0$  and  $y < 1$

1 : description

- 6 : { 1 : separates variables  
2 : antiderivatives  
1 : constant of integration  
1 : uses initial condition  
1 : solves for  $y$   
0/1 if  $y$  is not exponential

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables