

AP[®] CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 6

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let

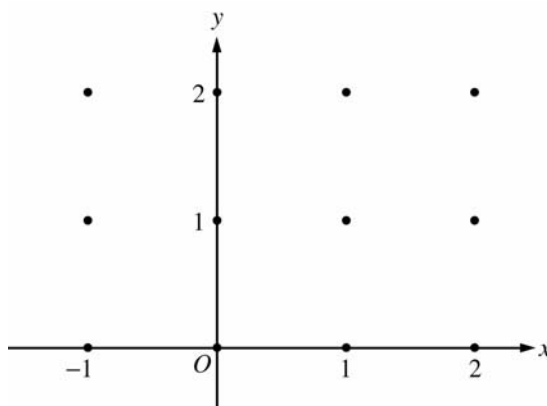
$y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

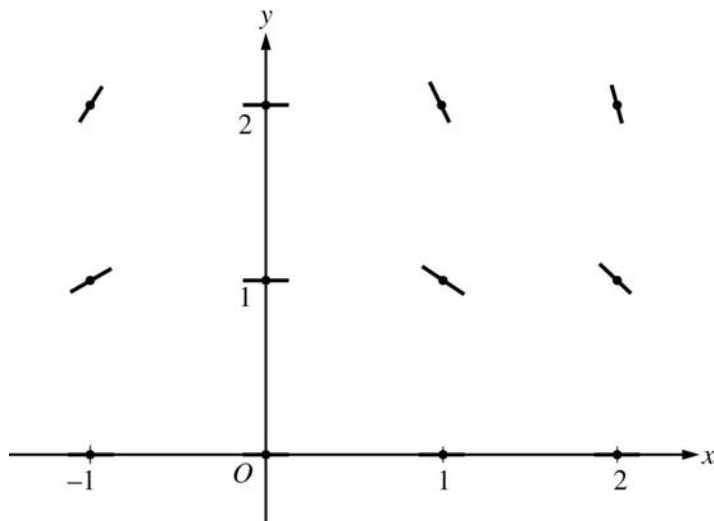
(Note: Use the axes provided in the test booklet.)

- (b) Write an equation for the line tangent to the graph of f at $x = -1$.

- (c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.



(a)



- (b) Slope = $\frac{-(-1)4}{2} = 2$
 $y - 2 = 2(x + 1)$

- (c) $\frac{1}{y^2} dy = -\frac{x}{2} dx$
 $-\frac{1}{y} = -\frac{x^2}{4} + C$
 $-\frac{1}{2} = -\frac{1}{4} + C; C = -\frac{1}{4}$
 $y = \frac{1}{\frac{x^2}{4} + \frac{1}{4}} = \frac{4}{x^2 + 1}$

2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

1 : equation

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables