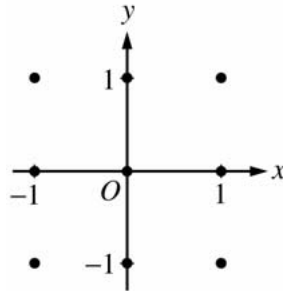


AP[®] CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 5

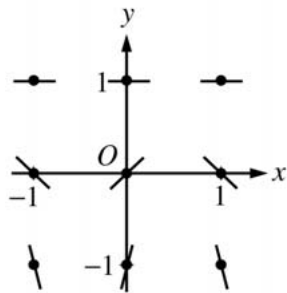
Consider the differential equation $\frac{dy}{dx} = (y - 1)^2 \cos(\pi x)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
 (Note: Use the axes provided in the exam booklet.)



- (b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c .
 (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

(a)



- (b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

(c)
$$\frac{1}{(y - 1)^2} dy = \cos(\pi x) dx$$

$$-(y - 1)^{-1} = \frac{1}{\pi} \sin(\pi x) + C$$

$$\frac{1}{1 - y} = \frac{1}{\pi} \sin(\pi x) + C$$

$$1 = \frac{1}{\pi} \sin(\pi) + C = C$$

$$\frac{1}{1 - y} = \frac{1}{\pi} \sin(\pi x) + 1$$

$$\frac{\pi}{1 - y} = \sin(\pi x) + \pi$$

$$y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \text{ for } -\infty < x < \infty$$

2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

1 : $c = 1$

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables