

1998 AP Calculus AB Scoring Guidelines

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.
- (a) Find the slope of the graph of f at the point where $x = 1$.
- (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
- (c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.
- (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$

$$\left. \frac{dy}{dx} \right|_{\substack{x=1 \\ y=4}} = \frac{3+1}{2 \cdot 4} = \frac{4}{8} = \frac{1}{2}$$

(b) $y - 4 = \frac{1}{2}(x - 1)$

$$f(1.2) - 4 \approx \frac{1}{2}(1.2 - 1)$$

$$f(1.2) \approx 0.1 + 4 = 4.1$$

(c) $2y \, dy = (3x^2 + 1) \, dx$

$$\int 2y \, dy = \int (3x^2 + 1) \, dx$$

$$y^2 = x^3 + x + C$$

$$4^2 = 1 + 1 + C$$

$$14 = C$$

$$y^2 = x^3 + x + 14$$

$$y = \sqrt{x^3 + x + 14} \text{ is branch with point } (1, 4)$$

$$f(x) = \sqrt{x^3 + x + 14}$$

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer

2 { 1: equation of tangent line
1: uses equation to approximate $f(1.2)$

5 { 1: separates variables
1: antiderivative of dy term
1: antiderivative of dx term
1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions
1: solves for y
0/1 if solving a linear equation in y
0/1 if no constant of integration

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x , y , or dy/dx before antidifferentiation

1: answer, from student's solution to the given differential equation in (c)