Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

(a) $e^{2y}\,dy = 3x^2\,dx$

$\dfrac{1}{2}e^{2y} = x^3 + C_1$

$e^{2y} = 2x^3 + C$

$y = \dfrac{1}{2}\ln\left(2x^3 + C\right)$

$\dfrac{1}{2} = \dfrac{1}{2}\ln(0 + C); \quad C = e$

$y = \dfrac{1}{2}\ln(2x^3 + e)$

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- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(0) = \dfrac{1}{2}$
- 1 : solves for y
- Note: 0/1 if y is not a logarithmic function of x

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

$x^3 > -\dfrac{1}{2}e$

$x > \left(-\dfrac{1}{2}e\right)^{1/3} = -\left(\dfrac{1}{2}e\right)^{1/3}$

Range: $-\infty < y < \infty$

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- 1 : $2x^3 + e > 0$
- 1 : domain
  - Note: 0/1 if 0 is not in the domain
- 1 : range

Note: 0/3 if y is not a logarithmic function of x