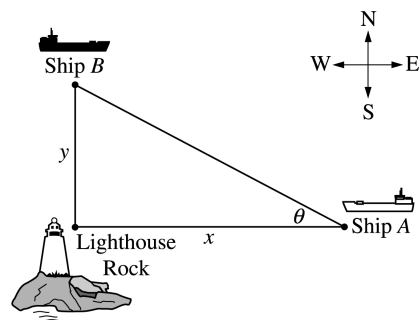


AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 6

Ship *A* is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship *B* is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship *A* and Lighthouse Rock at time t , and let y be the distance between Ship *B* and Lighthouse Rock at time t , as shown in the figure above.



- (a) Find the distance, in kilometers, between Ship *A* and Ship *B* when $x = 4$ km and $y = 3$ km.
- (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.
- (c) Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 4$ km and $y = 3$ km.

(a) Distance = $\sqrt{3^2 + 4^2} = 5$ km

(b) $r^2 = x^2 + y^2$

$$2r \frac{dr}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

or explicitly:

$$r = \sqrt{x^2 + y^2}$$

$$\frac{dr}{dt} = \frac{1}{2\sqrt{x^2 + y^2}} \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right)$$

At $x = 4$, $y = 3$,

$$\frac{dr}{dt} = \frac{4(-15) + 3(10)}{5} = -6 \text{ km/hr}$$

(c) $\tan \theta = \frac{y}{x}$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{\frac{dy}{dt} x - \frac{dx}{dt} y}{x^2}$$

At $x = 4$ and $y = 3$, $\sec \theta = \frac{5}{4}$

$$\frac{d\theta}{dt} = \frac{16}{25} \left(\frac{10(4) - (-15)(3)}{16} \right)$$

$$= \frac{85}{25} = \frac{17}{5} \text{ radians/hr}$$

1 : answer

4 { 1 : expression for distance
 2 : differentiation with respect to t
 < -2 > chain rule error
 1 : evaluation

4 { 1 : expression for θ in terms of x and y
 2 : differentiation with respect to t
 < -2 > chain rule, quotient rule, or
 transcendental function error
 note: 0/2 if no trig or inverse trig
 function
 1 : evaluation