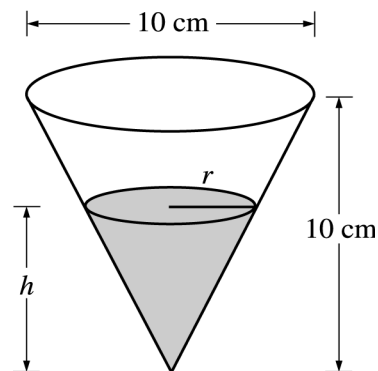


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Question 5

A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth  $h$  is changing at the constant rate of  $-\frac{3}{10}$  cm/hr.



(The volume of a cone of height  $h$  and radius  $r$  is given by  $V = \frac{1}{3}\pi r^2 h$ .)

- Find the volume  $V$  of water in the container when  $h = 5$  cm. Indicate units of measure.
- Find the rate of change of the volume of water in the container, with respect to time, when  $h = 5$  cm. Indicate units of measure.
- Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

(a) When  $h = 5$ ,  $r = \frac{5}{2}$ ;  $V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi \text{ cm}^3$

(b)  $\frac{r}{h} = \frac{5}{10}$ , so  $r = \frac{1}{2}h$   
 $V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3$ ;  $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$   
 $\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi \text{ cm}^3/\text{hr}$

OR

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt}\right); \frac{dr}{dt} = \frac{1}{2} \frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=5, r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$$

$$= -\frac{15}{8}\pi \text{ cm}^3/\text{hr}$$

(c)  $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} = -\frac{3}{40}\pi h^2$   
 $= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10} \cdot \text{area}$   
 The constant of proportionality is  $-\frac{3}{10}$ .

1 :  $V$  when  $h = 5$

5 :  $\left\{ \begin{array}{l} 1 : r = \frac{1}{2}h \text{ in (a) or (b)} \\ \\ V \text{ as a function of one variable} \\ \text{in (a) or (b)} \\ \\ \text{OR} \\ \\ \frac{dr}{dt} \\ \\ 2 : \frac{dV}{dt} \\ < -2 > \text{ chain rule or product rule error} \\ \\ 1 : \text{ evaluation at } h = 5 \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{ shows } \frac{dV}{dt} = k \cdot \text{area} \\ \\ 1 : \text{ identifies constant of} \\ \text{proportionality} \end{array} \right.$

units of  $\text{cm}^3$  in (a) and  $\text{cm}^3/\text{hr}$  in (b)

1 : correct units in (a) and (b)