

1992 AB6
Solution

$$(a) \frac{dV}{dt} = \frac{k}{r}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{k}{r} = 4\pi r^2 \frac{dr}{dt}$$

$$k dt = 4\pi r^3 dr$$

$$kt + C = \pi r^4$$

$$\text{At } t = 0, r = 1, \text{ so } C = \pi$$

$$\text{At } t = 15, r = 2, \text{ so } 15k + \pi = 16\pi, k = \pi$$

$$\pi r^4 = \pi t + \pi$$

$$r = \sqrt[4]{t+1}$$

$$(b) \text{ At } t = 0, r = 1, \text{ so } V(0) = \frac{4}{3}\pi$$

$$27V(0) = 27\left(\frac{4}{3}\pi\right) = 36\pi$$

$$36\pi = \frac{4}{3}\pi r^3$$

$$r = 3$$

$$\sqrt[4]{t+1} = 3$$

$$t = 80$$