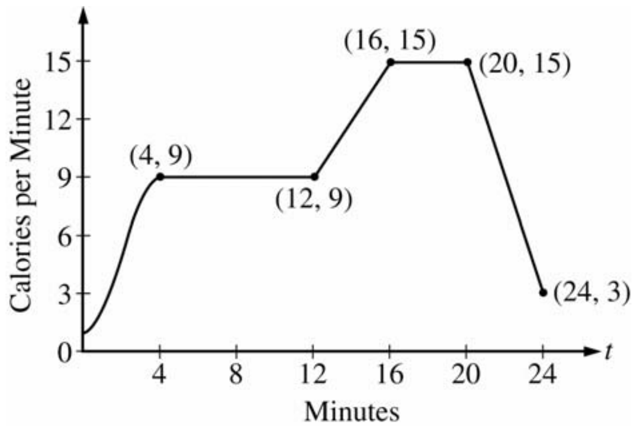
Question 4



The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.

(a) Find $f'(22)$. Indicate units of measure.

(b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

(c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

(d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

(a) $f'(22) = \dfrac{15 - 3}{20 - 24} = -3$ calories/min/min

1 : $f'(22)$ and units

(b) f is increasing on $[0, 4]$ and on $[12, 16]$.

On $(12, 16)$, $f'(t) = \dfrac{15 - 9}{16 - 12} = \dfrac{3}{2}$ since f has constant slope on this interval.

On $(0, 4)$, $f'(t) = -\dfrac{3}{4}t^2 + 3t$ and

$f''(t) = -\dfrac{3}{2}t + 3 = 0$ when $t = 2$. This is where f' has a maximum on $[0, 4]$ since $f'' > 0$ on $(0, 2)$ and $f'' < 0$ on $(2, 4)$.

On $[0, 24]$, f is increasing at its greatest rate when $t = 2$ because $f'(2) = 3 > \dfrac{3}{2}$.

4 :
- 1 : f' on $(0, 4)$
- 1 : shows f' has a max at $t = 2$ on $(0, 4)$
- 1 : shows for $12 < t < 16$, $f'(t) < f'(2)$
- 1 : answer

(c) $\displaystyle\int_6^{18} f(t)\,dt = 6(9) + \frac{1}{2}(4)(9 + 15) + 2(15)$

$= 132$ calories

2 :
- 1 : method
- 1 : answer

(d) We want $\dfrac{1}{12}\displaystyle\int_6^{18} (f(t) + c)\,dt = 15.$

This means $132 + 12c = 15(12)$. So, $c = 4$.

OR

Currently, the average is $\dfrac{132}{12} = 11$ calories/min.

Adding c to $f(t)$ will shift the average by c.
So $c = 4$ to get an average of 15 calories/min.

2 :
- 1 : setup
- 1 : value of c