

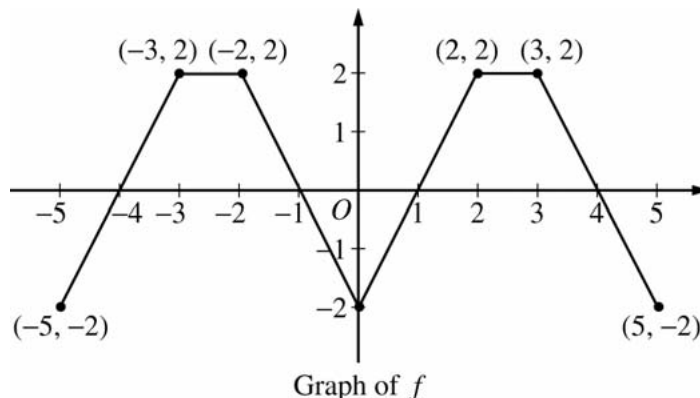
**AP[®] CALCULUS AB
2006 SCORING GUIDELINES**

Question 3

The graph of the function f shown above consists of six line segments. Let g be the function given by

$$g(x) = \int_0^x f(t) dt.$$

- (a) Find $g(4)$, $g'(4)$, and $g''(4)$.
 (b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



- (c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

(a) $g(4) = \int_0^4 f(t) dt = 3$

$$g'(4) = f(4) = 0$$

$$g''(4) = f'(4) = -2$$

- (b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

- (c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x .

$$g(10) = 2g(5) = 4$$

$$\begin{aligned} g(108) &= \int_0^{105} f(t) dt + \int_{105}^{108} f(t) dt \\ &= 21g(5) + g(3) = 44 \end{aligned}$$

$$g'(108) = f(108) = f(3) = 2$$

An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$$

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

$$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$