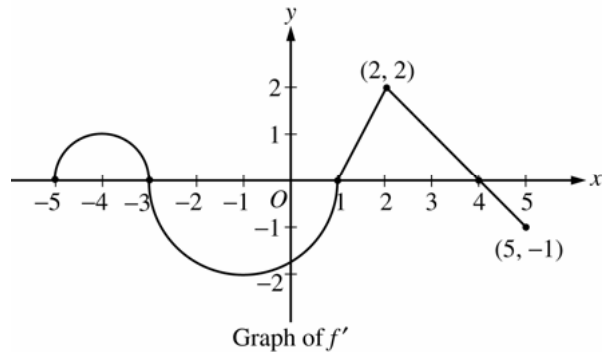


AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.



- (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
- (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
- (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
- (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3, 1, 4$
 f' changes from positive to negative at -3 and 4 .
 Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

2 : { 1 : x-values
 1 : justification

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4, -1$, and 2 . Thus, the graph of f has points of inflection when $x = -4, -1$, and 2 .

2 : { 1 : x-values
 1 : justification

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

2 : { 1 : intervals
 1 : explanation

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

3 : { 1 : identifies $x = 1$ as a candidate
 1 : considers endpoints
 1 : value and explanation

$$f(-5) = 3 + \int_1^{-5} f'(x) dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^5 f'(x) dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.