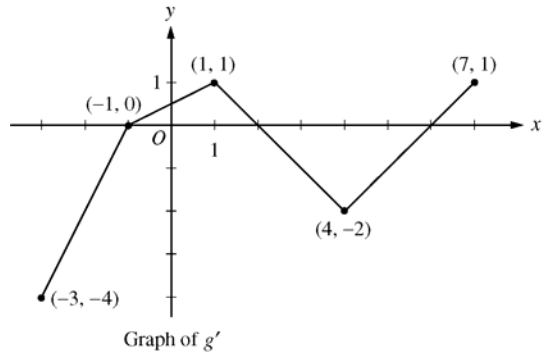


AP[®] CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown above for $-3 \leq x \leq 7$.



- (a) Find the x -coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.
- (b) Find the absolute maximum value of g on the interval $-3 \leq x \leq 7$. Justify your answer.
- (c) Find the average rate of change of $g(x)$ on the interval $-3 \leq x \leq 7$.
- (d) Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

- (a) g' changes from increasing to decreasing at $x = 1$;
 g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

- (b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x) dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$

$$g(2) = 5$$

$$g(7) = 5 + \int_2^7 g'(x) dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \leq x \leq 7$ is $\frac{15}{2}$.

- (c) $\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$

- (d) $\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$$

$$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$$

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$$