

1998 AP Calculus AB Scoring Guidelines

6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

(b) Write an equation of each horizontal tangent line to the curve.

(c) The line through the origin with slope -1 is tangent to the curve at point P . Find the x - and y -coordinates of point P .

(a) $6y^2 \frac{dy}{dx} + 6x^2 \frac{dy}{dx} + 12xy - 24x + 6 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$$

$$\frac{dy}{dx} = \frac{24x - 12xy}{6x^2 + 6y^2 + 6} = \frac{4x - 2xy}{x^2 + y^2 + 1}$$

(b) $\frac{dy}{dx} = 0$

$$4x - 2xy = 2x(2 - y) = 0$$

$$x = 0 \text{ or } y = 2$$

$$\text{When } x = 0, \quad 2y^3 + 6y = 1; \quad y = 0.165$$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

(c) $y = -x$ is equation of the line.

$$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$$

$$-8x^3 - 12x^2 - 6x - 1 = 0$$

$$x = -1/2, \quad y = 1/2$$

-or-

$$\frac{dy}{dx} = -1$$

$$4x - 2xy = -x^2 - y^2 - 1$$

$$4x + 2x^2 = -x^2 - x^2 - 1$$

$$4x^2 + 4x + 1 = 0$$

$$x = -1/2, \quad y = 1/2$$

2 { 1: implicit differentiation
1: verifies expression for $\frac{dy}{dx}$

4 { 1: sets $\frac{dy}{dx} = 0$
1: solves $\frac{dy}{dx} = 0$
1: uses solutions for x to find equations of horizontal tangent lines
1: verifies which solutions for y yield equations of horizontal tangent lines

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x, y)/h(x, y) = 0$ with solutions for both x and y

3 { 1: $y = -x$
1: substitutes $y = -x$ into equation of curve
1: solves for x and y

-or-

3 { 1: sets $\frac{dy}{dx} = -1$
1: substitutes $y = -x$ into $\frac{dy}{dx}$
1: solves for x and y

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)