

Consider the curve given by  $xy^2 - x^3y = 6$ .

- (a) Show that  $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ .
- (b) Find all points on the curve whose  $x$ -coordinate is 1, and write an equation for the tangent line at each of these points.
- (c) Find the  $x$ -coordinate of each point on the curve where the tangent line is vertical.

(a)  $y^2 + 2xy \frac{dy}{dx} - 3x^2y - x^3 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) When  $x = 1$ ,  $y^2 - y = 6$   
 $y^2 - y - 6 = 0$   
 $(y - 3)(y + 2) = 0$   
 $y = 3, y = -2$

At  $(1, 3)$ ,  $\frac{dy}{dx} = \frac{9 - 9}{6 - 1} = 0$

Tangent line equation is  $y = 3$

At  $(1, -2)$ ,  $\frac{dy}{dx} = \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$

Tangent line equation is  $y + 2 = 2(x - 1)$

(c) Tangent line is vertical when  $2xy - x^3 = 0$

$$x(2y - x^2) = 0 \text{ gives } x = 0 \text{ or } y = \frac{1}{2}x^2$$

There is no point on the curve with  $x$ -coordinate 0.

When  $y = \frac{1}{2}x^2$ ,  $\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

$$2 \left\{ \begin{array}{l} 1 : \text{implicit differentiation} \\ 1 : \text{verifies expression for } \frac{dy}{dx} \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 1 : y^2 - y = 6 \\ 1 : \text{solves for } y \\ 2 : \text{tangent lines} \end{array} \right.$$

Note: 0/4 if not solving an equation of the form  $y^2 - y = k$

$$3 \left\{ \begin{array}{l} 1 : \text{sets denominator of } \frac{dy}{dx} \text{ equal to 0} \\ 1 : \text{substitutes } y = \frac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \text{into the equation for the curve} \\ 1 : \text{solves for } x\text{-coordinate} \end{array} \right.$$