

AP[®] CALCULUS AB
2001 SCORING GUIDELINES

Question 6

The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

- (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.
- (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

(a)
$$\begin{aligned} \frac{d^2y}{dx^2} &= 2y \frac{dy}{dx} (6 - 2x) - 2y^2 \\ &= 2y^3(6 - 2x)^2 - 2y^2 \end{aligned}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\left(3, \frac{1}{4}\right)} = 0 - 2\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$$

(b)
$$\frac{1}{y^2} dy = (6 - 2x) dx$$

$$-\frac{1}{y} = 6x - x^2 + C$$

$$-4 = 18 - 9 + C = 9 + C$$

$$C = -13$$

$$y = \frac{1}{x^2 - 6x + 13}$$

$$3 : \left\{ \begin{array}{l} 2 : \frac{d^2y}{dx^2} \\ < -2 > \text{product rule or} \\ & \text{chain rule error} \\ 1 : \text{value at } \left(3, \frac{1}{4}\right) \end{array} \right.$$

$$6 : \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(3) = \frac{1}{4} \\ 1 : \text{solves for } y \end{array} \right.$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables