

AP[®] CALCULUS AB
2004 SCORING GUIDELINES

Question 4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

- (a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.
- (b) Show that there is a point P with x -coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y -coordinate of P .
- (c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.

(a) $2x + 8yy' = 3y + 3xy'$
 $(8y - 3x)y' = 3y - 2x$
 $y' = \frac{3y - 2x}{8y - 3x}$

2 : $\begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{solves for } y' \end{cases}$

(b) $\frac{3y - 2x}{8y - 3x} = 0$; $3y - 2x = 0$

When $x = 3$, $3y = 6$
 $y = 2$

$3^2 + 4 \cdot 2^2 = 25$ and $7 + 3 \cdot 3 \cdot 2 = 25$

Therefore, $P = (3, 2)$ is on the curve and the slope is 0 at this point.

3 : $\begin{cases} 1 : \frac{dy}{dx} = 0 \\ 1 : \text{shows slope is 0 at } (3, 2) \\ 1 : \text{shows } (3, 2) \text{ lies on curve} \end{cases}$

(c) $\frac{d^2y}{dx^2} = \frac{(8y - 3x)(3y' - 2) - (3y - 2x)(8y' - 3)}{(8y - 3x)^2}$

At $P = (3, 2)$, $\frac{d^2y}{dx^2} = \frac{(16 - 9)(-2) - (3y - 2x)(8y' - 3)}{(16 - 9)^2} = -\frac{2}{7}$.

Since $y' = 0$ and $y'' < 0$ at P , the curve has a local maximum at P .

4 : $\begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{value of } \frac{d^2y}{dx^2} \text{ at } (3, 2) \\ 1 : \text{conclusion with justification} \end{cases}$